

MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 18-03-2021 (MORNING SHIFT) | TIME : (9.00 am to 12.00 pm)

Duration 3 Hours | Max. Marks : 300

**QUESTION
&
SOLUTIONS**

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. An oil drop of radius 2 mm with a density 3g cm^{-3} is held stationary under a constant electric field $3.55 \times 10^5 \text{ V m}^{-1}$ in the Millikan's oil drop experiment. What is the number of excess electrons that the oil drop will possess ?

(consider $g = 9.81 \text{ m/s}^2$)

- (1) 48.8×10^{11} (2) 1.73×10^{10} (3) 17.3×10^{10} (4) 1.73×10^{12}

Ans. (2)

Sol. $qE = Mg$

$$neE = \rho \left(\frac{4}{3} \pi r^3 \right) \times g$$

$$n \times 1.6 \times 10^{-19} \times 3.55 \times 10^5$$

$$= 3 \times 10^3 \times \frac{4}{3} \times \pi \times (2 \times 10^{-3})^3 \times 9.81$$

$$n = 173 \times 10^{(3-9-5+19)}$$

$$n = 1.73 \times 10^{10}$$

2. Match List-I with List-II.

List-I

- (a) 10 km height over earth's surface
 (b) 70 km height over earth's surface
 (c) 180 km height over earth's surface
 (d) 270 km height over earth's surface

List-II

- (i) Thermosphere
 (ii) Mesosphere
 (iii) Stratosphere
 (iv) Troposphere
- (1) (a)–(iv), (b)–(iii), (c)–(ii), (d)–(i)
 (2) (a)–(i), (b)–(iv), (c)–(iii), (d)–(ii)
 (3) (a)–(iii), (b)–(ii), (c)–(i), (d)–(iv)
 (4) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)

Ans. (1)

Sol. Order of atmosphere stratification from bottom Troposphere, stratosphere, Mesosphere, Thermosphere

(a) → (iv)

(b) → (iii)

(c) → (ii)

(d) → (i)

3. Imagine that the electron in a hydrogen atom is replaced by a muon (μ). The mass of muon particle is 207 times that of an electron and charge is equal to the charge of an electron. The ionization potential of this hydrogen atom will be :-

- (1) 13.6 eV (2) 2815.2 eV (3) 331.2 eV (4) 27.2 eV

Ans. (2)

Sol. $E \propto \frac{1}{r}$ $r \propto \frac{1}{m}$

$E \propto m$

Ionization potential = $13.6 \times \frac{(\text{Mass}_\mu) \text{eV}}{(\text{Mass}_e)}$

= $13.6 \times 207 \text{ eV} = 2815.2 \text{ eV}$

4. A plane electromagnetic wave of frequency 100 MHz is travelling in vacuum along the x-direction. At a particular point in space and time, $\vec{B} = 2.0 \times 10^{-8} \hat{k} \text{ T}$. (where, \hat{k} is unit vector along z-direction) What is \vec{E} at this point ?

- (1) $0.6 \hat{j} \text{ V/m}$ (2) $6.0 \hat{k} \text{ V/m}$ (3) $6.0 \hat{j} \text{ V/m}$ (4) $0.6 \hat{k} \text{ V/m}$

Ans. (3)

Sol. $E = BC = 6$

(Dir. of wave) $\parallel (\vec{E} \times \vec{B})$

$\hat{i} = \hat{j} \times \hat{k}$

$\vec{E} = 6 \hat{j} \text{ V/m}$

5. A thin circular ring of mass M and radius r is rotating about its axis with an angular speed ω . Two particles having mass m each are now attached at diametrically opposite points. The angular speed of the ring will become :

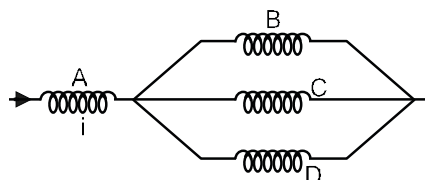
- (1) $\omega \frac{M}{M+2m}$ (2) $\omega \frac{M+2m}{M}$ (3) $\omega \frac{M}{M+2m}$ (4) $\omega \frac{M-2m}{M+2m}$

Ans. (3)

Sol. Using conservation of angular momentum $(Mr^2)\omega = (Mr^2 + 2mr^2)\omega'$

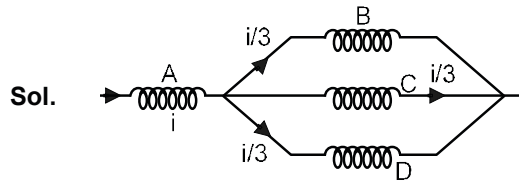
$\omega' = \frac{M\omega}{M+2m}$

6. Four identical long solenoids A, B, C and D are connected to each other as shown in the figure. If the magnetic field at the center of A is 3T, the field at the center of C would be : (Assume that the magnetic field is confined within the volume of respective solenoid).



- (1) 12T (2) 6T (3) 9T (4) 1T

Ans. (4)



$$\phi \propto i$$

$$\Rightarrow B \propto i$$

$$\text{so, field at centre of } C = \frac{3}{3} - IT$$

7. The time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{\ell}{g}}$. The measured value of the length of pendulum is 10 cm known to a 1mm accuracy. The time for 200 oscillations of the pendulum is found to be 100 second using a clock of 1s resolution. The percentage accuracy in the determination of 'g' using this pendulum is 'x'. The value of 'x' to the nearest integer is:-

- (1) 2% (2) 3% (3) 5% (4) 4%

Ans. (2)

Sol. $g = \frac{4\pi^2 \ell}{T^2}$

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2 \frac{\Delta T}{T} = \frac{0.1}{10} + 2 \left(\frac{1}{200} \right)$$

$$\frac{\Delta g}{g} = \frac{1}{100} + \frac{1}{50}$$

$$\frac{\Delta g}{g} \times 100 = 3\%$$

8. A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time 't' is proportional to :-

- (1) $t^{2/3}$ (2) $t^{3/2}$ (3) t (4) $t^{1/2}$

Ans. (2)

Sol. $P = C$

$$FV = C$$

$$M \frac{dV}{dt} V = C$$

$$\frac{V^2}{2} \propto t$$

$$V \propto t^{1/2}$$

$$\frac{dx}{dt} \propto t^{1/2}$$

$$x \propto t^{3/2}$$

9. What will be the average value of energy along one degree of freedom for an ideal gas in thermal equilibrium at a temperature T ? (k_B is Boltzmann constant)

- (1) $\frac{1}{2}k_B T$ (2) $\frac{2}{3}k_B T$ (3) $\frac{3}{2}k_B T$ (4) $k_B T$

Ans. (1)

Sol. Energy associated with each degree of freedom per molecule = $\frac{1}{2}k_B T$

10. A radioactive sample disintegrates via two independent decay processes having half lives $T_{1/2}^{(1)}$ and $T_{1/2}^{(2)}$ respectively. The effective half-life $T_{1/2}$ of the nuclei is :

- (1) None of the above (2) $T_{1/2} = T_{1/2}^{(1)} + T_{1/2}^{(2)}$ (3) $T_{1/2} = \frac{T_{1/2}^{(1)} T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$ (4) $T_{1/2} = \frac{T_{1/2}^{(1)} + T_{1/2}^{(2)}}{T_{1/2}^{(1)} - T_{1/2}^{(2)}}$

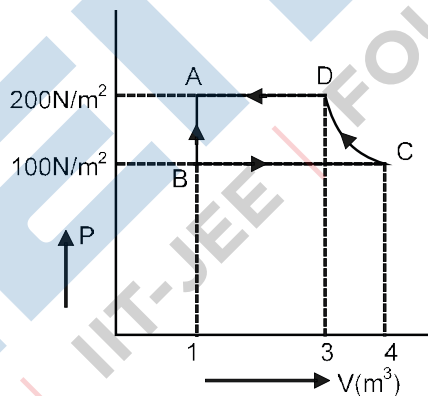
Ans. (3)

Sol. $\lambda_{eq} = \lambda_1 + \lambda_2$

$$\frac{1}{T_{1/2}} = \frac{1}{T_{1/2}^{(1)}} + \frac{1}{T_{1/2}^{(2)}}$$

$$T_{1/2} = \frac{T_{1/2}^{(1)} T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$$

11. The P-V diagram of a diatomic ideal gas system going under cyclic process as shown in figure. The work done during an adiabatic process CD is (use $\gamma = 1.4$) :



- (1) -500 J (2) -400 J (3) 400 J (4) 200 J

Ans. (1)

Sol. Adiabatic process is from C to D

$$\begin{aligned} WD &= \frac{P_2 V_2 - P_1 V_1}{1 - \gamma} \\ &= \frac{P_D V_D - P_C V_C}{1 - \gamma} \\ &= \frac{200(3) - (100)(4)}{1 - 1.4} \\ &= -500 \text{ J} \end{aligned}$$

12. In Young's double slit arrangement, slits are separated by a gap of 0.5 mm, and the screen is placed at a distance of 0.5 m from them. The distance between the first and the third bright fringe formed when the slits are illuminated by a monochromatic light of 5890 Å is :-

- (1) 1178×10^{-9} m (2) 1178×10^{-6} m (3) 1178×10^{-12} m (4) 5890×10^{-7} m

Ans. (2)

Sol.
$$\beta = \frac{\lambda D}{d} = \frac{5890 \times 10^{-10} \times 0.5}{0.5 \times 10^{-3}}$$

$$= 589 \times 10^{-6} \text{ m}$$

Distance between first and third bright fringe

is $2\beta = 2 \times 589 \times 10^{-6} \text{ m}$
 $= 1178 \times 10^{-6} \text{ m}$

13. A particle is travelling 4 times as fast as an electron. Assuming the ratio of de-Broglie wavelength of a particle to that of electron is 2 : 1, the mass of the particle is :-

- (1) $\frac{1}{16}$ times the mass of e^- (2) 8 times the mass of e^-
 (3) 16 times the mass of e^- (4) $\frac{1}{8}$ times the mass of e^-

Ans. (4)

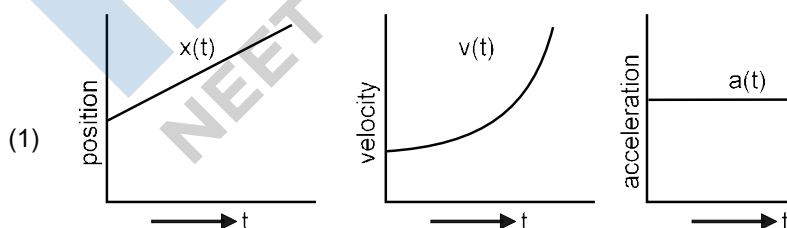
Sol.
$$\lambda = \frac{h}{p}$$

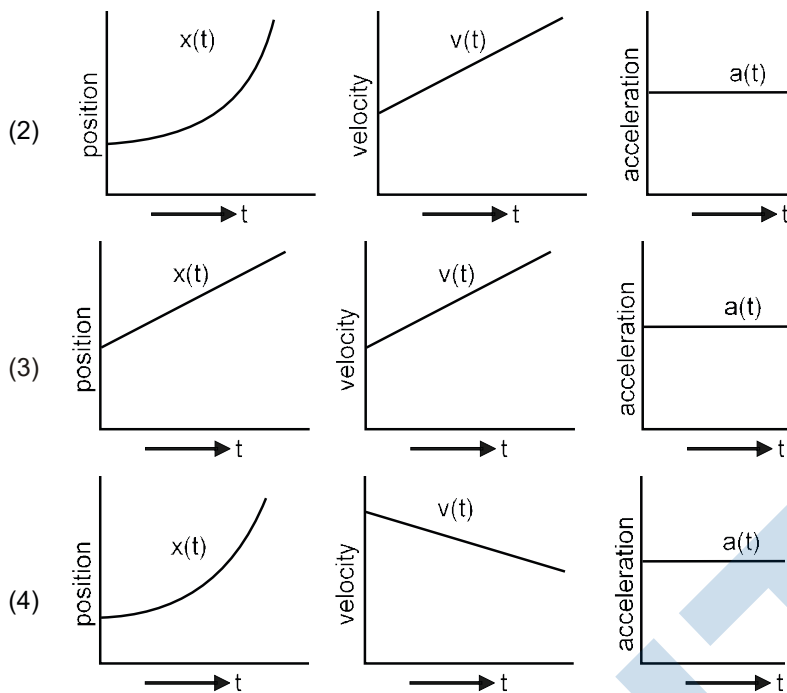
$$\frac{\lambda_p}{\lambda_e} = \frac{p_e}{p_p} = \frac{m_e v_e}{m_p v_p}$$

$$2 = \frac{m_e \left(\frac{v_e}{4v_e} \right)}{m_p}$$

$$\therefore m_p = \frac{m_e}{8}$$

14. The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by :





Ans. (2)

Sol. Option (2) represent correct graph for particle moving with constant acceleration, as for constant acceleration velocity time graph is straight line with positive slope and x-t graph should be an opening upward parabola.

15. In the experiment of Ohm's law, a potential difference of 5.0 V is applied across the end of a conductor of length 10.0 cm and diameter of 5.00 mm. The measured current in the conductor is 2.00 A. The maximum permissible percentage error in the resistivity of the conductor is :-

- (1) 3.9 (2) 8.4 (3) 7.5 (4) 3.0

Ans. (1)

Sol. $R = \frac{\rho \ell}{A} = \frac{V}{I}$

$$\rho = \frac{AV}{I\ell} = \frac{\pi d^2 V}{4I\ell} \quad \left(A = \frac{\pi d^2}{4} \right)$$

$$\therefore \frac{\Delta \rho}{\rho} = \frac{2\Delta d}{d} + \frac{\Delta V}{V} + \frac{\Delta I}{I} + \frac{\Delta \ell}{\ell}$$

$$\frac{\Delta \rho}{\rho} = 2 \left(\frac{0.01}{5.00} \right) + \frac{0.1}{5.0} + \frac{0.01}{2.00} + \frac{0.1}{10.0}$$

$$\frac{\Delta \rho}{\rho} = 0.004 + 0.02 + 0.005 + 0.01$$

$$\frac{\Delta \rho}{\rho} = 0.039$$

$$\% \text{ error} = \frac{\Delta \rho}{\rho} \times 100 = 0.039 \times 100 = 3.90\%$$

- 16.** In a series LCR resonance circuit, if we change the resistance only, from a lower to higher value :
- (1) The bandwidth of resonance circuit will increase.
 - (2) The resonance frequency will increase.
 - (3) The quality factor will increase.
 - (4) The quality factor and the resonance frequency will remain constant.

Ans. (1)

Sol. Bandwidth = R/L

Bandwidth $\propto R$

So bandwidth will increase

- 17.** An AC source rated 220 V, 50 Hz is connected to a resistor. The time taken by the current to change from its maximum to the rms value is :
- (1) 2.5 ms
 - (2) 25 ms
 - (3) 2.5 s
 - (4) 0.25 ms

Ans. (1)

Sol. $i = i_0 \cos(\omega t)$

$i = i_0$ at $t = 0$

$$i = \frac{i_0}{\sqrt{2}} \text{ at } \omega t = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4(2\pi f)} = \frac{1}{8f}$$

$$t = \frac{1}{400} = 2.5 \text{ ms}$$

- 18.** Your friend is having eye sight problem. She is not able to see clearly a distant uniform window mesh and it appears to her as nonuniform and distorted. The doctor diagnosed the problem as :
- (1) Astigmatism
 - (2) Myopia with Astigmatism
 - (3) Presbyopia with Astigmatism
 - (4) Myopia and hypermetropia

Ans. (2)

Sol. If distant objects are blurry then problem is Myopia.

If objects are distorted then problem is Astigmatism

- 19.** A loop of flexible wire of irregular shape carrying current is placed in an external magnetic field. Identify the effect of the field on the wire.
- (1) Loop assumes circular shape with its plane normal to the field.
 - (2) Loop assumes circular shape with its plane parallel to the field.
 - (3) Wire gets stretched to become straight.
 - (4) Shape of the loop remains unchanged.

Ans. (1)

Sol. Every part ($d\ell$) of the wire is pulled by force $i(d\ell)B$ acting perpendicular to current & magnetic field giving it a shape of circle.

20. The time period of a satellite in a circular orbit of radius R is T . The period of another satellite in a circular orbit of radius $9R$ is :

(1) $9T$

(2) $27T$

(3) $12T$

(4) $3T$

Ans. (2)

Sol. $T^2 \propto R^3$

$$\left(\frac{T'}{T}\right)^2 = \left(\frac{9R}{R}\right)^3$$

$$T'^2 = T^2 \times 9^3$$

$$T' = T \times 3^3$$

$$T' = 27T$$

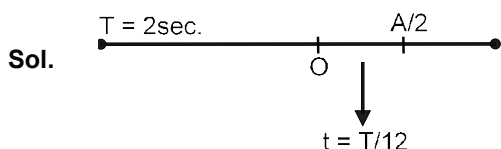
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Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. A particle performs simple harmonic motion with a period of 2 second. The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is $\frac{1}{s}$ s. The value of 'a' to the nearest integer is _____.

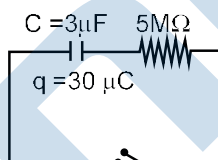
Ans. (6)



$$t = \frac{2}{12} = \frac{1}{6}$$

∴ Correct answer = 6.00

2. The circuit shown in the figure consists of a charged capacitor of capacity $3 \mu\text{F}$ and a charge of $30 \mu\text{C}$. At time $t = 0$, when the key is closed, the value of current flowing through the $5 \text{ M}\Omega$ resistor is 'x' μA . The value of 'x' to the nearest integer is _____.

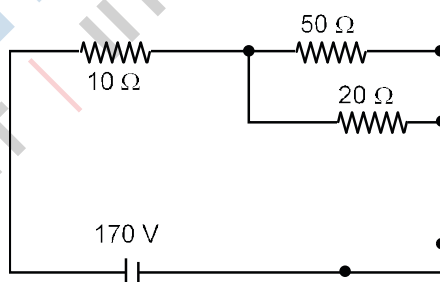


Ans. (2)

Sol.
$$i_0 = \frac{V}{R} = \frac{30/3}{5 \times 10^6} = 2 \times 10^{-6}$$

∴ Ans. = 2.00

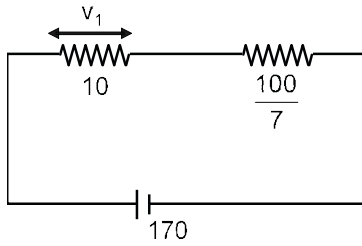
3. The voltage across the 10Ω resistor in the given circuit is x volt.



The value of 'x' to the nearest integer is _____.

Ans. (70)

Sol.
$$R_{\text{eq}} = \frac{50 \times 20}{70} = \frac{100}{7}$$



$$R_{eq} = \frac{170}{7}$$

$$V_1 = \left[\frac{170}{7} \right] \times 10 = 70V$$

Ans. = 70.00

4. Two separate wires A and B are stretched by 2 mm and 4 mm respectively, when they are subjected to a force of 2 N. Assume that both the wires are made up of same material and the radius of wire B is 4 times that of the radius of wire A. The length of the wires A and B are in the ratio of a : b. Then a/b can be expressed as 1/x where x is _____ .

Ans. (32)

Sol. For A $\frac{E}{\pi r^2} = y \frac{2mm}{a}$ (1)

For B $\frac{E}{\pi \cdot 16r^2} = y \frac{4mm}{b}$ (2)

∴ (1)/(2)

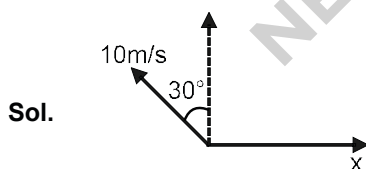
$$16 = \frac{2b}{4a}$$

$$\frac{a}{b} = \frac{1}{32}$$

∴ Answer = 32

5. A person is swimming with a speed of 10 m/ s at an angle of 120° with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is 'x' m/s. The value of 'x' to the nearest integer is _____ .

Ans. (5)



$$10 \sin 30^\circ = x$$

$$x = 5 \text{ m/s}$$

6. A parallel plate capacitor has plate area 100 m^2 and plate separation of 10 m . The space between the plates is filled up to a thickness 5 m with a material of dielectric constant of 10 . The resultant capacitance of the system is 'x' pF.

The value of $\epsilon_0 = 8.85 \times 10^{-12} \text{ F.m}^{-1}$.

The value of 'x' to the nearest integer is _____.

Ans. (161)

Sol. $A = 100 \text{ m}^2$

Using $C = \frac{k \epsilon_0 A}{d}$

$$C_1 = \frac{10 \epsilon_0 (100)}{5}$$

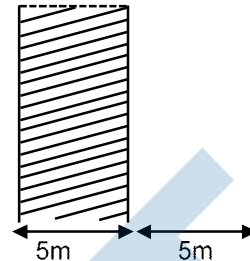
$$= 200 \epsilon_0$$

$$C_2 = \frac{\epsilon_0 (100)}{5} = 20 \epsilon_0$$

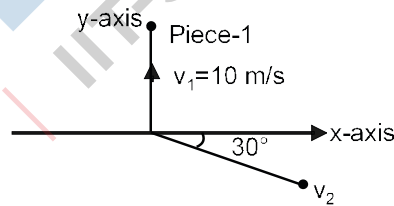
C_1 & C_2 are in series so $C_{\text{equiv.}} = \frac{C_1 C_2}{C_1 + C_2}$

$$= \frac{400 \epsilon_0}{220}$$

$$= 160.9 \times 10^{-12} \approx 161 \text{ pF}$$



7. A ball of mass 10 kg moving with a velocity $10\sqrt{3} \text{ m/s}$ along the x-axis, hits another ball of mass 20 kg which is at rest. After the collision, first ball comes to rest while the second ball disintegrates into two equal pieces. One piece starts moving along y-axis with a speed of 10 m/s . The second piece starts moving at an angle of 30° with respect to the x-axis. The velocity of the ball moving at 30° with x-axis is $x \text{ m/s}$. The configuration of pieces after collision is shown in the figure below. The value of x to the nearest integer is _____.



Ans. (20)

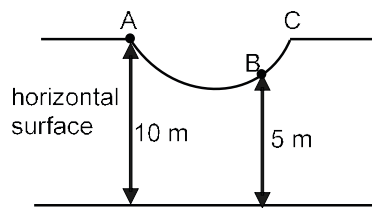
Sol. Let velocity of 2nd fragment is \vec{v} then by conservation of linear momentum

$$10(10\sqrt{3})\hat{i} = (10)(10\hat{j}) + 10\vec{v}$$

$$\Rightarrow \vec{v} = 10\sqrt{3}\hat{i} - 10\hat{j}$$

$$|\vec{v}| = \sqrt{300 + 100} = \sqrt{400} = 200 \text{ m/s}$$

8. As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s. (Take $g = 10 \text{ m/s}^2$)
The value of 'x' to the nearest integer is _____.



Ans. (10)

Sol. Using work energy theorem,

$$W_g = \Delta \text{K.E.}$$

$$(10)(g)(5) = \frac{1}{2}(10)v^2 - 0$$

$$v = 10 \text{ m/s}$$

9. An npn transistor operates as a common emitter amplifier with a power gain of 10^6 . The input circuit resistance is 100Ω and the output load resistance is $10 \text{ K}\Omega$. The common emitter current gain ' β ' will be _____.

Ans. (100)

Sol. $10^6 = \beta^2 \times \frac{R_o}{R_i}$

$$10^6 = \beta^2 \times \frac{10^4}{10^2}$$

$$\beta^2 = 10^4 \Rightarrow \beta = 100$$

10. A bullet of mass 0.1 kg is fired on a wooden block to pierce through it, but it stops after distance of 50 cm into it. If the velocity of bullet before hitting the wood is 10 m/s and it slows down with uniform deceleration, then the magnitude of effective retarding force on the bullet is 'x' N. The value of 'x' to the nearest integer is _____.

Ans. (10)

Sol. $v^2 = u^2 + 2as$

$$0 = (10)^2 + 2(-a)\left(\frac{1}{2}\right)$$

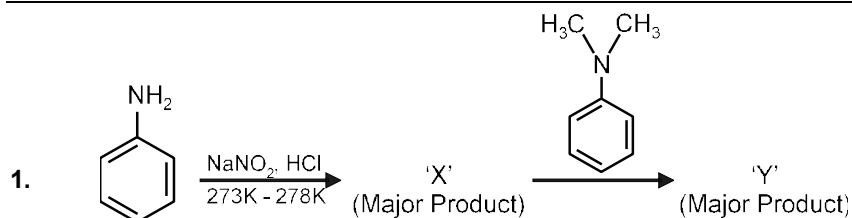
$$a = 100 \text{ m/s}^2$$

$$F = ma = (0.1)(100) = 10 \text{ N}$$

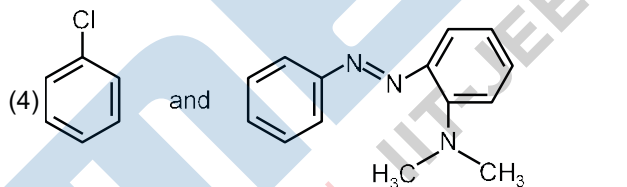
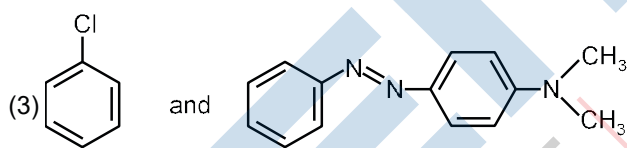
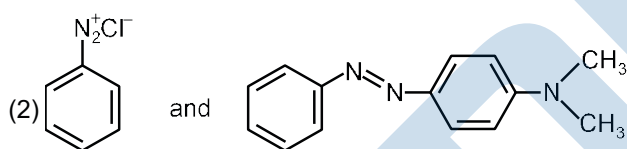
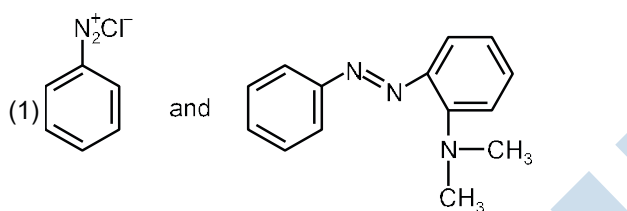
PART B : CHEMISTRY

Single Choice Type

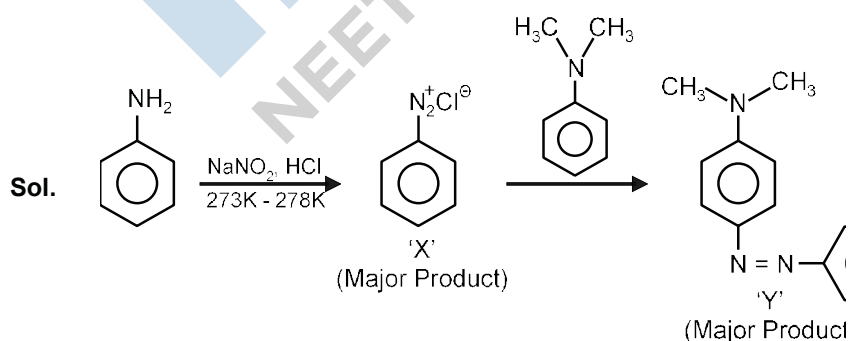
This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.



Considering the above reaction, X and Y respectively are :



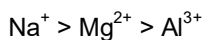
Ans. (2)



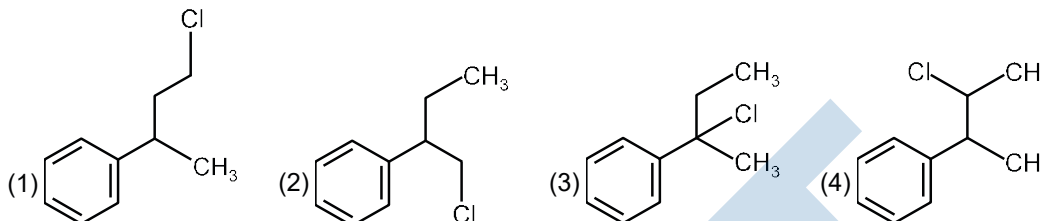
2. The ionic radius of Na^+ ions is 1.02 \AA . The ionic radii (in \AA) of Mg^{2+} and Al^{3+} , respectively, are-
 (1) 1.05 and 0.99 (2) 0.72 and 0.54 (3) 0.85 and 0.99 (4) 0.68 and 0.72

Ans. (2)

Sol. The ionic radii order is



3. Reaction of Grignard reagent, $\text{C}_2\text{H}_5\text{MgBr}$ with $\text{C}_8\text{H}_8\text{O}$ followed by hydrolysis gives compound "A" which reacts instantly with Lucas reagent to give compound B, $\text{C}_{10}\text{H}_{13}\text{Cl}$. The Compound B is :



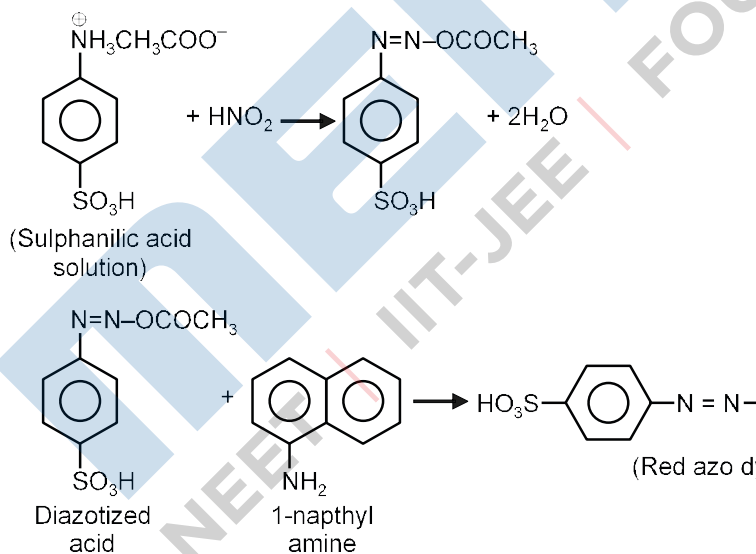
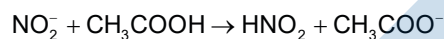
Ans. (3)

4. Reagent, 1-naphthylamine and sulphanilic acid in acetic acid is used for the detection of

- (1) N_2O (2) NO_3^- (3) NO (4) NO_2^-

Ans. (4)

Sol. For detection of NO_2^- , the following test is used.



5. A non-reducing sugar "A" hydrolyses to give two reducing mono saccharides. Sugar A is-
 (1) Fructose (2) Galactose (3) Glucose (4) Sucrose

Ans. (4)

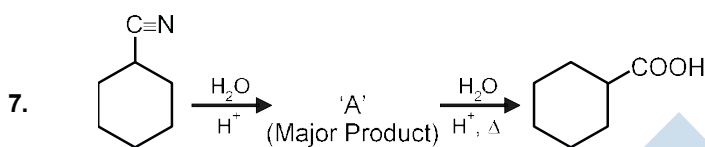
Sol. $\text{Sucrose} \xrightarrow{\text{H}_2\text{O}} \text{Glucose} + \text{Fructose}$
 (Non reducing sugar) (Reducing sugar) (Reducing sugar)

6. Match the list -I with list - II

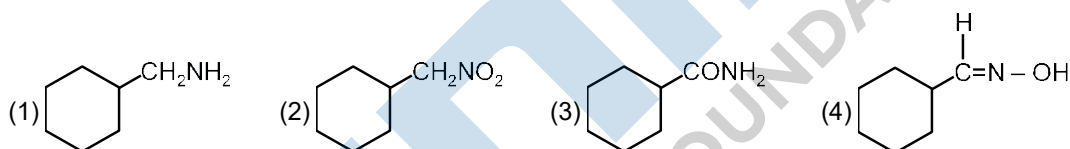
List-I (Class of Drug)	List-II (Example)
(a) Antacid	(i) Novestrol
(b) Artificial sweetener	(ii) Cimetidine
(c) Antifertility	(iii) Valium
(d) Tranquilizers	(iv) Alitame
(1) (a) – (ii), (b) – (iv), (c) – (i), (d) – (iii)	(2) (a) – (iv), (b) – (i), (c) – (ii), (d) – (iii)
(3) (a) – (iv), (b) – (iii), (c) – (i), (d) – (ii)	(4) (a) – (ii), (b) – (iv), (c) – (iii), (d) – (i)

Ans. (1)

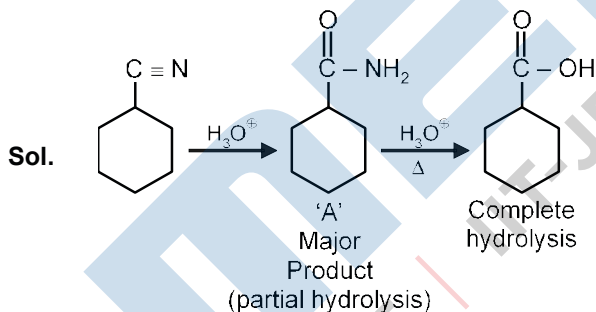
Sol. (a) Antacid : Cimetidine (b) Artificial Sweetener : Alitame
(c) Antifertility : Novestrol (d) Tranquilizers : Valium



Consider the above chemical reaction and identify product "A"



Ans. (3)



8. Match List-I with List-II

List-I	List-II
(a) Chlorophyll	(i) Ruthenium
(b) Vitamin-B ₁₂	(ii) Platinum
(c) Anticancer drug	(iii) Cobalt
(d) Grubbs catalyst	(iv) Magnesium

Choose the most appropriate answer from the options given below :

(a) a-iii, b-ii, c-iv, d-i (b) a-iv, b-iii, c-ii, d-i (c) a-iv, b-iii, c-i, d-ii (d) a-iv, b-ii, c-iii, d-i

Ans. (2)

Sol. Chlorophyll is a coordination compound of magnesium.

Vitamin B-12, cyanocobalamin is a coordination compound of cobalt.

Cisplatin is used as an anti-cancer drug and is a coordination compound of platinum.

Grubbs catalyst is a compound of Ruthenium.

9. Match List-I with List-II :

List-I (Chemicals)	List-II (Use / Preparation / Constituent)
(a) Alcoholic potassium hydroxide	(i) Electrodes in batteries
(b) Pd/ BaSO ₄	(ii) Obtained by addition reaction
(c) BHC (Benzene hexachloride)	(iii) Used for β-elimination reaction
(d) Polyacetylene	(iv) Lindlar's catalyst

Choose the most appropriate match :

(1) a-ii, b-i, c-iv, d-iii

(2) a-iii, b-iv, c-ii, d-i

(3) a-iii, b-i, c-iv, d-ii

(4) a-ii, b-iv, c-i, d-iii

Ans. (2)

Sol. (a) Alcoholic potassium hydroxide → used for β-elimination

(b) Pd/ BaSO₄ → Lindlar's catalyst

(c) BHC (Benzene hexachloride) → Obtained by addition reactions

(d) Polyacetylene → Electrodes in batteries

10. The statements that are TRUE :

(A) Methane leads to both global warming and photochemical smog

(B) Methane is generated from paddy fields

(C) Methane is a stronger global warming gas than CO₂

(D) Methane is a part of reducing smog

Choose the most appropriate answer from the options given below :

(1) (A), (B), (C) only

(2) (A) and (B) only

(3) (B), (C), (D) only

(4) (A), (B), (D) only

Ans. (1)

Sol. Methane leads to both global warming & photochemical smog.

Methane is generated in large amounts from paddy fields.

CO₂ can be absorbed by photosynthesis, or by formation of acid rain etc., while no such activities are there for methane.

Hence methane is stronger global warming gas than CH₄.

Methane is not a part of reducing smog.

11. Match List-I with List-II

List-I	List-II
(a) $\text{Ca}(\text{OCl})_2$	(i) Antacid
(b) $\text{CaSO}_4 \cdot \frac{1}{2}\text{H}_2\text{O}$	(ii) Cement
(c) CaO	(iii) Bleach
(d) CaCO_3	(iv) Plaster of paris

Choose the most appropriate answer from the options given below :

- (1) a-i, b-iv, c-iii, d-ii (2) a-iii, b-ii, c-iv, d-i (3) a-iii, b-iv, c-ii, d-l (4) a-iii, b-ii, c-i, d-iv

Ans. (3)

Sol. $\text{Ca}(\text{OCl})_2$ is Bleach.

$\text{CaSO}_4 \cdot \frac{1}{2}\text{H}_2\text{O}$ is plaster of paris.

CaCO_3 is used as an antacid.

CaO is major component of cement.

12. Compound with molecular formula $\text{C}_3\text{H}_6\text{O}$ can show :

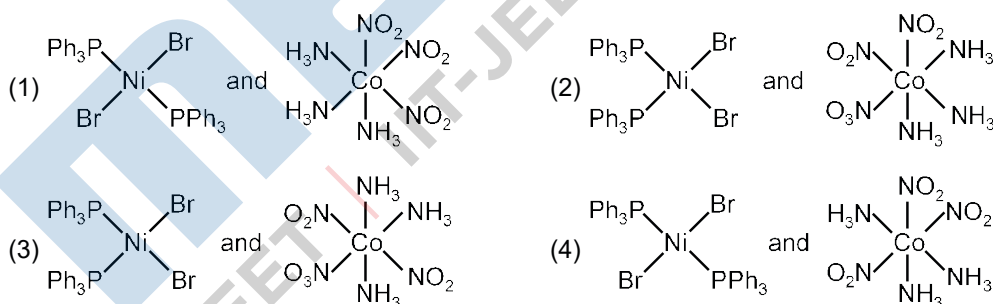
- (1) Positional isomerism (2) Both positional isomerism and metamerism
 (3) Metamerism (4) Functional group isomerism

Ans. (4)

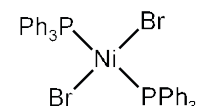
Sol. $\text{C}_3\text{H}_6\text{O} \Rightarrow \text{CH}_3 - \text{CH}_2 - \text{CH} = \text{O}$ & $\text{CH}_3 - \overset{\text{O}}{\parallel}{\text{C}} - \text{CH}_3$

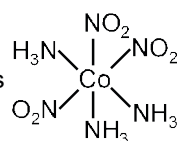
They are functional group isomerism.

13. The correct structures of $\text{trans} - [\text{NiBr}_2(\text{PPh}_3)_2]$ and meridional- $[\text{Co}(\text{NH}_3)_3(\text{NO}_2)_3]$, respectively, are



Ans. (4)

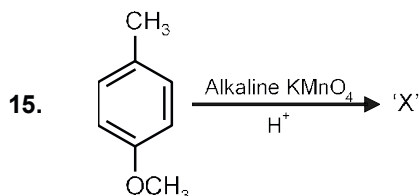
Sol. $\text{trans} - [\text{Ni Br}_2(\text{PPh}_3)_2]$ is 

meridional - $[\text{Co}(\text{NH}_3)_3(\text{NO}_2)_3]$ is 

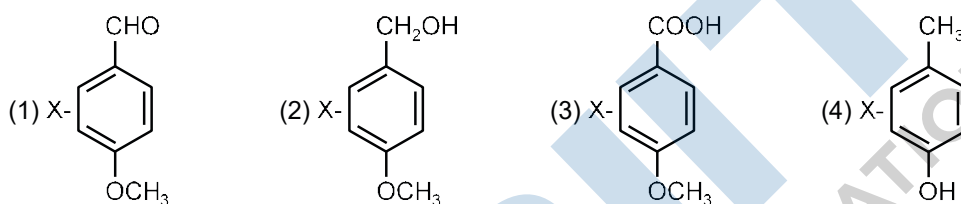
14. A certain orbital has no angular nodes and two radial nodes. The orbital is :
 (1) 2s (2) 3s (3) 3p (4) 2p

Ans. (2)

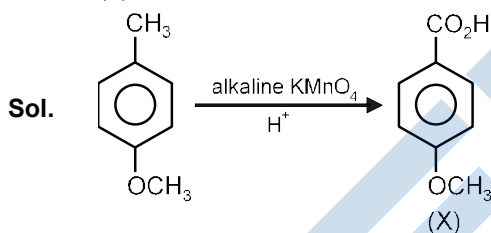
Sol. $l = 0 \Rightarrow$'s' orbital
 $n - l - 1 = 2$
 $n - 1 = 2$
 $n = 3$



Considering the above chemical reaction, identify the product "X" :



Ans. (3)



16. Match List-I with List-II

List-I (process)

- (a) Deacon's process
 (b) Contact process
 (c) Cracking of hydrocarbons
 (d) Hydrogenation of vegetable oils

List-II (catalyst)

- (i) ZSM-5
 (ii) CuCl_2
 (iii) Particles 'Ni'
 (iv) V_2O_5

Choose the most appropriate answer from the options given below -

- (1) a-ii, b-iv, c-i, d-iii (2) a-i, b-iii, c-ii, d-iv (3) a-iii, b-i, c-iv, d-ii (4) a-iv, b-ii, c-i, d-iii

Ans. (1)

Sol. In manufacture of H_2SO_4 (contact process), V_2O_5 is used as a catalyst.

Ni catalysts enables the hydrogenation of fats.

CuCl_2 is used as catalyst in Deacon's process.

ZSM-5 used as catalyst in cracking of hydrocarbons.

17. Given below are two statements : One is labelled as Assertion A and the other labelled as reason R

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. AX is a covalent diatomic molecule where A and X are second row elements of periodic table. Based on Molecular orbital theory, the bond order of AX is 2.5. The total number of electrons in AX is _____.
(Round off to the Nearest Integer).

Ans. (15)

Sol. AX is a covalent diatomic molecule.

The molecule is NO.

Total no. of electrons is 15.

2. In order to prepare a buffer solution of pH 5.74, sodium acetate is added to acetic acid. If the concentration of acetic acid in the buffer is 1.0 M, the concentration of sodium acetate in the buffer is _____ M.

[Given : pKa (acetic acid) = 4.74]

Ans. (10)

Sol. $pH = pKa + \log \frac{[CB]}{[WA]}$

$$5.74 = 4.74 + \log \frac{[CB]}{1}$$

$$\Rightarrow [CB] = 10 \text{ M}$$

3. $2 \text{ NO}(g) + \text{Cl}_2(g) \rightleftharpoons 2 \text{ NOCl}(s)$

This reaction was studied at -10°C and the following data was obtained

run	$[\text{NO}]_0$	$[\text{Cl}_2]_0$	r_0
1	0.10	0.10	0.18
2	0.10	0.20	0.35
3	0.20	0.20	1.40

$[\text{NO}]_0$ and $[\text{Cl}_2]_0$ are the initial concentrations and r_0 is the initial reaction rate. The overall order of the reaction is _____.

Ans. (3)

Sol. $r = k[\text{NO}]^m [\text{Cl}_2]^n$

$$= k(0.1)^m (0.1)^n \dots (1)$$

$$= k(0.1)^m (0.2)^n \dots (2)$$

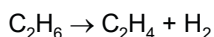
$$= k(0.2)^m (0.2)^n \dots (3)$$

$$n = 1$$

$$m = 2$$

$$m + n = 3$$

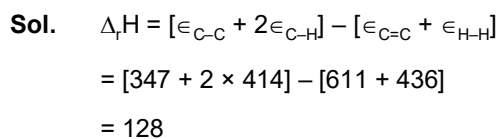
4. For the reaction



the reaction enthalpy $\Delta_r H = \text{_____ kJ mol}^{-1}$.

[Given : Bond enthalpies in kJ mol^{-1} : C-C : 347, C=C : 611; C-H : 414, H-H : 436]

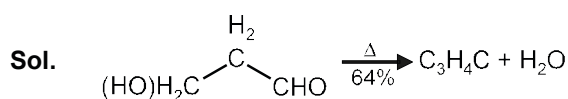
Ans. (128)



5. _____ grams of 3-Hydroxy propanal (MW=74) must be dehydrated to produce 7.8 g of acrolein (MW = 56) ($\text{C}_3\text{H}_4\text{O}$) if the percentage yield is 64.

[Given : Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u]

Ans. (16)



$$\frac{x}{74} \text{ mol} \qquad \frac{x}{74} \times 0.64 = \frac{7.8}{56}$$

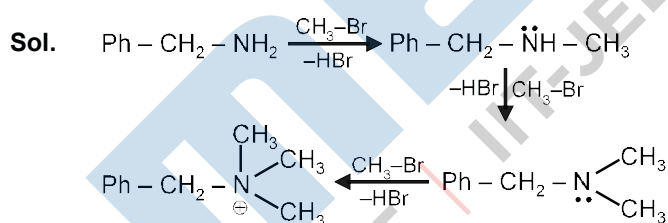
$$x = 16.10$$

□ 16.00

6. A reaction of 0.1 mole of Benzylamine with bromomethane gave 23 g of Benzyl trimethyl ammonium bromide. The number of moles of bromomethane consumed in this reaction are $n \times 10^{-1}$, when $n = \text{_____}$.

(Given : Atomic masses : C : 12.0 u, H : 1.0 u, N : 14.0 u, Br : 80.0 u)

Ans. (3)



no of moles = 3

7. The total number of unpaired electrons present in the complex $\text{K}_3[\text{Cr}(\text{oxalate})_3]$ is _____.

Ans. (3)

Sol. $\text{K}_3[\text{Cr}(\text{oxalate})_3]$

Chromium is in +3 oxidation state.

Number of unpaired electrons in Cr^{+3} will be 3.

8. 2 molal solution of a weak acid HA has a freezing point of 3.885°C . The degree of dissociation of this acid is _____ $\times 10^{-3}$.

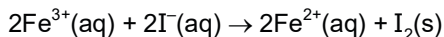
[Given : Molal depression constant of water = $1.85 \text{ K kg mol}^{-1}$ Freezing point of pure water = 0°C]

Ans. (50)

Sol. $\Delta T_f = (1 + a) K_f \cdot m$

$$\alpha = 0.05 = 50 \times 10^{-3}$$

9. For the reaction

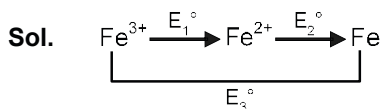


the magnitude of the standard molar free energy change, $\Delta_r G_m^0 = - \text{_____ kJ}$.

$$\left[\begin{array}{l} E_{\text{Fe}^{2+}/\text{Fe}(\text{s})}^0 = -0.440; \quad E_{\text{Fe}^{3+}/\text{Fe}(\text{s})}^0 = -0.036 \text{ V} \\ E_{\text{I}_2/2\text{I}^{-}}^0 = 0.539 \text{ V}; \quad F = 96500 \text{ C} \end{array} \right]$$

Ans. (46)

ZIGYAN Ans. (45)



$$E_1^0 + 2E_2^0 = 3E_3^0$$

$$E_1^0 = 3E_3^0 - 2E_2^0$$

$$= 3(-0.036) - 2(-0.44)$$

$$= +0.772 \text{ V}$$

$$E_{\text{cell}}^0 = E_{\text{Fe}^{3+}/\text{Fe}^{2+}}^0 + E_{\text{I}^{-}/\text{I}_2}^0 = 0.233$$

$$\Delta_r G^0 = -2 \times 96.5 \times 0.233 = -45 \text{ kJ}$$

10. Complete combustion of 3 g of ethane gives $x \times 10^{22}$ molecules of water. The value of x is _____.

[Use : $N_A = 6.023 \times 10^{23}$; Atomic masses in u : C : 12.0 ; O : 16.0 ; H : 1.0]

Ans. (18)



$$0.1 \text{ mol} \qquad 0.3 = 0.3 \times 6 \times 10^{23} = 18 \times 10^{22}$$

mol \qquad mol

$$\text{No. of molecules} = 0.3 \times 6.023 \times 10^{23} = 18.069 \times 10^{22}$$

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is :

(1) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

(2) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

(3) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

(4) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

Ans. (3)

Sol. $y^2 = 4ax + 4a^2$

differentiate with respect to x

$$\Rightarrow 2y \frac{dy}{dx} = 4a$$

$$\Rightarrow a = \left(\frac{y}{2} \frac{dy}{dx}\right)$$

so, required differential equation is

$$y^2 = \left(4 \times \frac{y}{2} \frac{dy}{dx}\right)x + 4\left(\frac{y}{2} \frac{dy}{dx}\right)^2$$

$$\Rightarrow y^2 \left(\frac{dy}{dx}\right)^2 + 2xy \left(\frac{dy}{dx}\right) - y^2 = 0$$

$$\Rightarrow y \left(\frac{dy}{dx}\right)^2 + 2x \left(\frac{dy}{dx}\right) - y = 0$$

2. The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is :

(1) 1

(2) 2

(3) 3

(4) 0

Ans. (2)

Sol. $3x + 4y = 9$

$$y = mx + 1$$

$$\Rightarrow 3x + 4mx + 4 = 9$$

$$\Rightarrow (3 + 4m)x = 5$$

$\Rightarrow x$ will be an integer when

$$3 + 4m = 5, -5, 1, -1$$

$$\Rightarrow m = \frac{1}{2}, -2, -\frac{1}{2}, -1$$

so, number of integral values of m is 2

3. Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \dots + a_{40}x^{40}$. then $a_1 + a_3 + a_5 + \dots + a_{37}$ is equal to
 (1) $2^{20}(2^{20} - 21)$ (2) $2^{19}(2^{20} - 21)$ (3) $2^{19}(2^{20} + 21)$ (4) $2^{20}(2^{20} + 21)$

Ans. (2)

Sol. $(1 + x + 2x^2)^{20} = a_0 + a_1x + \dots + a_{40}x^{40}$ put $x=1, -1$

$$\Rightarrow a_0 + a_1 + a_2 + \dots + a_{40} = 2^{20}$$

$$a_0 - a_1 + a_2 - \dots + a_{40} = 2^{20}$$

$$\Rightarrow a_1 + a_3 + \dots + a_{39} = \frac{4^{20} - 2^{20}}{2}$$

$$\Rightarrow a_1 + a_3 + \dots + a_{37} = 2^{39} - 2^{19} - a_{39}$$

$$\text{here } a_{39} = \frac{20!(2)^{19} \times 1}{19!} = 20 \times 2^{19}$$

$$\begin{aligned} \Rightarrow a_1 + a_3 + \dots + a_{37} &= 2^{19}(2^{20} - 1 - 20) \\ &= 2^{19}(2^{20} - 21) \end{aligned}$$

4. The solutions of the equation $\begin{vmatrix} 1 + \sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0, (0 < x < \pi)$, are

- (1) $\frac{\pi}{12}, \frac{\pi}{6}$ (2) $\frac{\pi}{6}, \frac{5\pi}{6}$ (3) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (D) $\frac{7\pi}{12}, \frac{11\pi}{12}$

Ans. (4)

Sol. $\begin{vmatrix} 1 + \sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0$

use $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow (2 + 4 \sin 2x) \begin{vmatrix} 1 & 1 & 1 \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0$$

$$\Rightarrow \sin 2x = -\frac{1}{2}$$

$$\Rightarrow 2x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$

$$x = \frac{\pi}{2} + \frac{\pi}{12}, \pi - \frac{\pi}{12}$$

5. Choose the correct statement about two circles whose equations are given below :

$$x^2 + y^2 - 10x - 10y + 41 = 0$$

$$x^2 + y^2 - 22x - 10y + 137 = 0$$

- (1) circles have same centre (2) circles have no meeting point
 (3) circles have only one meeting point (4) circles have two meeting points

Ans. (3)

Sol. $x^2 + y^2 - 10x - 10y + 41 = 0$

$A(5,5), R_1 = 3$

$x^2 + y^2 - 22x - 10y + 137 = 0$

$B(11,5), R_2 = 3$

$AB = 6 = R_1 + R_2$

Touch each other externally

⇒ circles have only one meeting point.

6. Let α, β, γ be the real roots of the equation, $x^3 + ax^2 + bx + c = 0$, ($a, b, c \in \mathbb{R}$ and $a, b \neq 0$). If the system of equations (in u, v, w) given by $\alpha u + \beta v + \gamma w = 0$, $\beta u + \gamma v + \alpha w = 0$; $\gamma u + \alpha v + \beta w = 0$ has non-trivial solution, then the value of $\frac{a^2}{b}$ is

(1) 5

(2) 3

(3) 1

(4) 0

Ans. (2)

Sol.
$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

⇒ $-(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \Sigma\alpha\beta) = 0$

⇒ $-(-a)(a^2 - 2b - b) = 0$

⇒ $a(a^2 - 3b) = 0$

⇒ $a^2 = 3b \Rightarrow \frac{a^2}{b} = 3$

7. The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}} dx$ is equal to

(where c is a constant of integration)

(1) $\frac{1}{2} \sin\sqrt{(2x-1)^2+5} + c$

(2) $\frac{1}{2} \cos\sqrt{(2x+1)^2+5} + c$

(3) $\frac{1}{2} \cos\sqrt{(2x-1)^2+5} + c$

(4) $\frac{1}{2} \sin\sqrt{(2x-1)^2+5} + c$

Ans. (1)

Sol.
$$\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{(2x-1)^2+5}} dx$$

$(2x-1)^2 + 5 = t^2$

$2(2x-1) 2dx = 2t dt$

$2\sqrt{t^2-5}dx = t dt$

$$\begin{aligned} \text{So } \int \frac{\sqrt{t^2 - 5} \cos t}{2\sqrt{t^2 - 5}} dt &= \frac{1}{2} \sin t + c \\ &= \frac{1}{2} \sin \sqrt{(2x - 1)^2 + 5} + c \end{aligned}$$

8. The equation of one of the straight lines which passes through the point (1, 3) and makes an angle $\tan^{-1}(\sqrt{2})$ with the straight line, $y + 1 = 3\sqrt{2}x$ is

- (1) $4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$ (2) $5\sqrt{2}x + 4y - (15 + 4\sqrt{2}) = 0$
 (3) $4\sqrt{2}x + 5y - 4\sqrt{2} = 0$ (4) $4\sqrt{2}x - 5y - (5 + 4\sqrt{2}) = 0$

Ans. (1)

Sol. $y = mx + c$

$$3 = m + c$$

$$\sqrt{2} = \left| \frac{m - 3\sqrt{2}}{1 + 3\sqrt{2}m} \right|$$

$$= 6m + \sqrt{2} = m - 3\sqrt{2}$$

$$= \sin = -4\sqrt{2} \rightarrow m = \frac{-4\sqrt{2}}{5}$$

$$= 6m - \sqrt{2} = m - 3\sqrt{2}$$

$$= 7m - 2\sqrt{2} \rightarrow m = \frac{2\sqrt{2}}{7}$$

According to options take $m = \frac{-4\sqrt{2}}{5}$

$$\text{So } y = \frac{-4\sqrt{2}x}{5} + \frac{3 + 4\sqrt{2}}{5}$$

$$4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$$

9. If $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{3x^3}$ is equal to L, then the value of (6L + 1) is

- (1) $\frac{1}{6}$ (2) $\frac{1}{2}$ (3) 6 (4) 2

Ans. (4)

Sol.
$$\lim_{x \rightarrow 0} \frac{\left(x + \frac{x^3}{3!} \dots\right) - \left(x - \frac{x^3}{3} \dots\right)}{3x^3} = \frac{1}{6}$$

So $6L + 1 = 2$

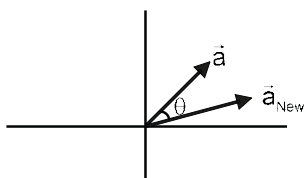
10. A vector \vec{a} has components $3p$ and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, \vec{a} has components $p + 1$ and $\sqrt{10}$, then a value of p is equal to:

- (1) 1 (2) $-\frac{5}{4}$ (3) $\frac{4}{5}$ (4) -1

Ans. (4)

Sol. $\vec{a}_{Old} = 3p\hat{i} + \hat{j}$

$\vec{a}_{New} = (p + 1)\hat{i} + \sqrt{10}\hat{j}$



$\Rightarrow |\vec{a}_{Old}| = |\vec{a}_{New}|$

$\Rightarrow ap^2 + 1 = p^2 + 2p + 1 + 10$

$8p^2 - 2p - 10 = 0$

$4p^2 - p - 5 = 0$

$(4p - 5)(p + 1) = 0 \rightarrow p = \frac{5}{4}, -1$

11. If the equation $a|z|^2 + \overline{\alpha z} + \alpha \overline{z} + d = 0$ represents a circle where a, d are real constants then which of the following condition is correct ?

- (1) $|\alpha|^2 - ad \neq 0$ (2) $|\alpha|^2 - ad > 0$ and $a \in \mathbb{R} - \{0\}$
 (3) $|\alpha|^2 - ad \geq 0$ and $a \in \mathbb{R}$ (4) $\alpha = 0, a, d \in \mathbb{R}^+$

Ans. (2)

Sol. $az\overline{z} + \alpha\overline{z} + \overline{\alpha z} + d = 0 \rightarrow$ Circle

centre = $\frac{-\alpha}{a}$ $2 = \sqrt{\frac{\alpha\overline{\alpha}}{a^2} - \frac{d}{a}} = \sqrt{\frac{a\overline{a} - ad}{a^2}}$

So $|\alpha|^2 - ad > 0$ & $a \in \mathbb{R} - \{0\}$

12. For the four circles M, N, O and P, following four equations are given :

Circle M : $x^2 + y^2 = 1$

Circle N : $x^2 + y^2 - 2x = 0$

Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P : $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a :

- (1) Rhombus (2) Square (3) Rectangle (4) Parallelogram

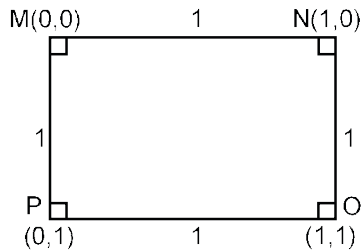
Ans. (2)

Sol. M : $x^2 + y^2 = 1$ (0,0)

N : $x^2 + y^2 - 2x = 0$ (1,0)

O : $x^2 + y^2 - 2x - 2y + 1 = 0$ (1,1)

P : $x^2 + y^2 - 2y = 0$ (0,1)



13. If α, β are natural numbers such that $100^\alpha - 199\beta = (100)(100) + (99)(101) + (98)(102) + \dots + (1)(199)$, then the slope of the line passing through (α, β) and origin is :

- (1) 540 (2) 550 (3) 530 (4) 510

Ans. (2)

Sol. $S = (100)(100) + (99)(101) + (98)(102) + \dots + (2)(198) + (1)(199)$

$$S = \sum_{x=0}^{99} (100-x)(100+x) = \sum 100^2 - x^2$$

$$= 100^3 - \frac{99 \times 100 \times 199}{6}$$

$\alpha = 3$ $\beta = 1650$

slope = $\frac{1650}{3} = 550$

14. The real valued function $f(x) = \frac{\text{cosec}^{-1} x}{\sqrt{x - [x]}}$, where $[x]$ denotes the greatest integer less than or equal to x , is defined for all x belonging to :

- (1) all reals except integers (2) all non-integers except the interval $[-1, 1]$
 (3) all integers except 0, -1, 1 (4) all reals except the interval $[-1, 1]$

Ans. (2)

Sol. $f(x) = \frac{\text{cosec}^{-1} x}{\sqrt{\{x\}}}$

Domain $\in (-\infty, -1] \cup [1, \infty)$

$\{x\} \neq 0$ so $x \neq$ integers

15. $\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + \dots + \frac{1}{(201)^2-1}$ is equal to

- (1) $\frac{101}{404}$ (2) $\frac{25}{101}$ (3) $\frac{101}{408}$ (4) $\frac{99}{400}$

Ans. (2)

Sol. $T_n = \frac{1}{(2n+1)^2 - 1} \frac{1}{(2n+2)2n} = \frac{1}{4n(n+1)}$
 $= \frac{(n+1) - n}{4n(n+1)} = \frac{1}{4} \left(\frac{1}{n} - \frac{1}{n+1} \right)$
 $S = \frac{1}{4} \left(1 - \frac{1}{101} \right) = \frac{1}{4} \left(\frac{100}{101} \right) = \frac{25}{101}$

16. If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions :

$f + g, f - g, f/g, g/f, g - f$ where $(f \pm g)(x) = f(x) \pm g(x), (f/g)(x) = \frac{f(x)}{g(x)}$

- (1) $0 < x < 1$ (2) $0 < x < 1$ (3) $0 < x < 1$ (4) $0 < x < 1$

Ans. (3)

Sol. $f(x) + g(x) = \sqrt{x} + \sqrt{1-x}$, domain $[0, 1]$

$f(x) - g(x) = \sqrt{x} - \sqrt{1-x}$, domain $[0, 1]$

$g(x) - f(x) = \sqrt{1-x} - \sqrt{x}$, domain $[0, 1]$

$\frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\sqrt{1-x}}$, domain $(0, 1)$

$\frac{g(x)}{f(x)} = \frac{\sqrt{1-x}}{\sqrt{x}}$, domain $(0, 1)$

So, common domain is $(0, 1)$

17. If $f(x) = \begin{cases} \frac{1}{|x|} & ; |x| \geq 1 \\ ax^2 + b & ; |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively :

- (1) $\frac{1}{2}, \frac{1}{2}$ (2) $\frac{1}{2}, -\frac{3}{2}$ (3) $\frac{5}{2}, -\frac{3}{2}$ (4) $-\frac{1}{2}, \frac{3}{2}$

Ans. (4)

Sol. $f(x) = \begin{cases} \frac{1}{|x|}, & |x| \geq 1 \\ ax^2 + b, & |x| < 1 \end{cases}$

at $x = 1$ function must be continuous

So, $1 = a + b$... (1)

differentiability at $x = 1$

$\left(-\frac{1}{x^2} \right)_{x=1} = (2ax)_{x=1}$

$$\Rightarrow -1 = 2a \Rightarrow a = -\frac{1}{2}$$

$$(1) \Rightarrow b = 1 + \frac{1}{2} = \frac{3}{2}$$

18. Let $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$

and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 3 \\ 0 & 1 & 2 \end{bmatrix}$. If $\text{Tr}(A)$ denote

sum of all diagonal elements of the matrix A, then $\text{Tr}(A) - \text{Tr}(B)$ has value equal to

(1) 1

(2) 2

(3) 0

(4) 3

Ans. (2)

Sol. $A + 2B = \begin{pmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{pmatrix} \dots(1)$

$$2A - B = \begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 3 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\Rightarrow 4A - 2B = \begin{pmatrix} 4 & -2 & 10 \\ 4 & -2 & 12 \\ 0 & 2 & 4 \end{pmatrix} \dots(2)$$

$$(1) + (2) \Rightarrow 5A = \begin{pmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix} \text{ and } 2A = \begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix}$$

$$\therefore B = \begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{pmatrix}$$

$$\text{tr } A = 1 - 1 + 1 = 1$$

$$\text{tr}(B) = -1$$

$$\text{tr}(A) = 1 \text{ and } \text{tr}(B) = -1$$

$$\therefore \text{tr}(A) - \text{tr}(B) = 2$$

19. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is:

- (1) 26664 (2) 122664 (3) 122234 (4) 22264

Ans. (1)

Sol. Digits are 1, 2, 2, 3

total distinct numbers $\frac{4!}{2!} = 12.$

total numbers when 1 at unit place is 3.

2 at unit place is 6

3 at unit place is 3.

$$\begin{aligned} \text{So, sum} &= (3 + 12 + 9) (10^3 + 10^2 + 10 + 1) \\ &= (1111) \times 24 \\ &= 26664 \end{aligned}$$

20. The value of $3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots \infty}}}}$ is equal to

- (1) $1.5 + \sqrt{3}$ (2) $2 + \sqrt{3}$ (3) $3 + 2\sqrt{3}$ (4) $4 + \sqrt{3}$

Ans. (1)

Sol. Let $x = 3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots \infty}}}}$

$$\text{So, } x = 3 + \frac{1}{4 + \frac{1}{x}} = 3 + \frac{x}{4x + 1}$$

$$\Rightarrow (x - 3) = \frac{x}{(4x + 1)}$$

$$\Rightarrow (4x + 1)(x - 3) = x$$

$$\Rightarrow 4x^2 - 12x + x - 3 = x$$

$$\Rightarrow 4x^2 - 12x - 3 = 0$$

$$x = \frac{12 \pm \sqrt{(12)^2 + 12 \times 4}}{2 \times 4} = \frac{12 \pm \sqrt{12(16)}}{8} = \frac{12 \pm 4 \times 2\sqrt{3}}{8} = \frac{3 \pm 2\sqrt{3}}{2}$$

$$x = \frac{3}{2} \pm \sqrt{3} = 1.5 \pm \sqrt{3}.$$

But only positive value is accepted

$$\text{So, } x = 1.5 + \sqrt{3}$$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is

Ans. (300)

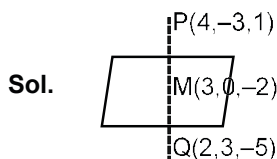
Sol. $3_ _ = 10 \times 10 = 100$

$_ 3 _ = 10 \times 10 = 100$

$_ _ 3 = 10 \times 10 = \frac{100}{300}$

2. Let the plane $ax + by + cz + d = 0$ bisect the line joining the points $(4, -3, 1)$ and $(2, 3, -5)$ at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is

Ans. (28)



Plane is $1(x - 3) - 3(y - 0) + 3(z + 2) = 0$

$x - 3y + 3z + 3 = 0$

$(a^2 + b^2 + c^2 + d^2)_{\min} = 28$

3. Let $f(x)$ and $g(x)$ be two functions satisfying $f(x^2) + g(4 - x) = 4x^3$ and $g(4 - x) + g(x) = 0$, then the value of $\int_{-4}^4 f(x)^2 dx$ is

Ans. (512)

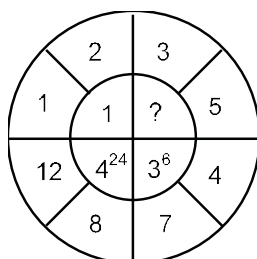
Sol. $I = 2 \int_0^4 f(x^2) dx$ {Even function}

$= 2 \int_0^4 (4x^3 - g(4 - x)) dx$

$= 2 \left(\frac{4x^4}{4} \Big|_0^4 - \int_0^4 g(4 - x) dx \right)$

$= 2(256 - 0) = 512$

4. The missing value in the following figure is



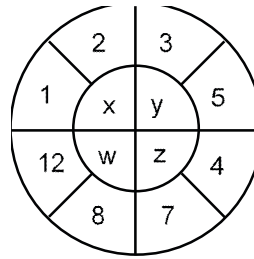
Ans. (4)

Sol. $x = (2 - 1)^{11} = 1$

$w = (12 - 8)^{41} = 4^{24}$

$z = (7 - 4)^{31} = 3^6$

hence $y = (5 - 3)^{21} = 2^2$



5. Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is

Ans. (6)

Sol. If $0, z_1, z_2$ are vertices of equilateral triangles

$\Rightarrow a^2 + z_1^2 + z_2^2 = 0(z_1 + z_2) + z_1 z_2$

$\Rightarrow (z_1 + z_2)^2 = 3z_1 z_2$

$\Rightarrow a^2 = 3 \times 12$

$\Rightarrow |a| = 6$

6. The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at unit distance from the point $(1, 2, 3)$ is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is

Ans. (4)

Sol. Let plane is $x - 2y + 2z + \lambda = 0$

distance from $(1, 2, 3) = 1$

$\Rightarrow \frac{|\lambda + 3|}{5} = 1 \Rightarrow \lambda = 0, -6$

$\Rightarrow a = 1, b = -2, c = 2, d = 0$ or 0

$b - d = -2$ or $2, c - a = 0$

$\Rightarrow k = 4$ or -2

7. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is _.

Ans. (35)

Sol. $\frac{\sum x_i}{25} = 40$ & $\frac{\sum x_i - 60 + N}{25} = 39$

Let age of newly appointed teacher is N

$\Rightarrow 1000 - 60 + N = 975$

$\Rightarrow N = 35$ years

8. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)} dx, (x \geq 0), f(0) = 0$ and $f(1) = \frac{1}{K}$, then the value of K is

Ans. (4)

Sol. $f(x) = \int \frac{(5x^8 + 7x^6) dx}{x^{14}(x^{-5} + x^{-7} + 2)^2}$

Let $x^{-5} + x^{-7} + 2 = t$

$(-5x^{-6} - 7x^{-8}) dx = dt$

$$\Rightarrow f(x) = \int -\frac{dt}{t^2} = \frac{1}{t} + c$$

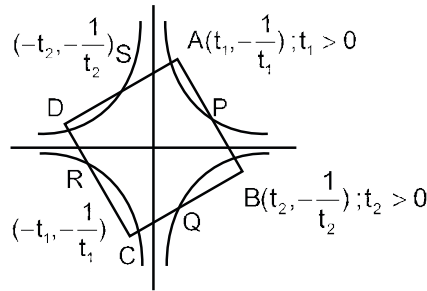
$$f(x) = \frac{x^7}{x^2 + 1 + 2x^7}$$

$$f(1) = \frac{1}{4}$$

9. A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is

Ans. (80)

Sol. $xy = 1, -1$



$$\frac{t_1 + t_2}{2} \cdot \frac{\frac{1}{t_1} - \frac{1}{t_2}}{2} = 1$$

$$\Rightarrow t_1^2 - t_2^2 = 4t_1t_2$$

$$\frac{1}{t_1^2} \times \left(-\frac{1}{t_2^2}\right) = -1 \Rightarrow t_1t_2 = 1$$

$$\Rightarrow (t_1t_2)^2 = 1 \Rightarrow t_1t_2 = 1$$

$$t_1^2 - t_2^2 = 4$$

$$\Rightarrow t_1^2 + t_2^2 = \sqrt{4^2 + 4} = 2\sqrt{5}$$

$$\Rightarrow t_1^2 = 2 + \sqrt{5} \Rightarrow \frac{1}{t_1^2} = \sqrt{5} - 2$$

$$\begin{aligned} AB^2 &= (t_1 - t_2)^2 + \left(\frac{1}{t_1} + \frac{1}{t_2}\right)^2 \\ &= 2\left(t_1^2 + \frac{1}{t_1^2}\right) = 4\sqrt{5} \Rightarrow \text{Area}^2 = 80 \end{aligned}$$

10. The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, \pi]$ is

Ans. (1)

Sol. If $\cot x > 0 \Rightarrow \frac{1}{\sin x} = 0$ (Not possible)

$$\text{If } \cot x < 0 \Rightarrow 2\cot x + \frac{1}{\sin x} = 0$$

$$\Rightarrow 2\cos x = -1$$

$$\Rightarrow x = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3} \text{ (reject)}$$